Absorbing prime-like Generalizations of prime ideals

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R always denotes a commutative ring with unity. In the last decades, several generalizations of prime ideals are established and studied; below are some of them:

1. In 2007, Ayman Badawi introduced the concept of **2-absorbing ideal**. He defined a non-zero proper ideal I of a ring R to be 2-absorbing if whenever $a, b, c \in R$ and $abc \in I$, then $ab \in I$ *or* $ac \in I$ *or* $bc \in I$ [4]. In 2011, The concept of 2-absorbing is generalized into **n-absorbing** by David F. Anderson and Ayman Badawi [1].
2. In 2019, Ayman Badawi and Ece Yetkin Celikel used another technique to generalize the prime ideals and introduced the **1-absorbing primary ideal**. They defined a non-zero proper ideal I of a ring R to be 1-absorbing primary if whenever $a, b, c \in R$ and $abc \in I$, then $ab \in I$ *or* $c \in \sqrt{I}$. It is clear that the 1-absorbing primary is also generalizes the 2-absorbing ideals. [2]
3. In 2020, A. Yassine, M. J. Nikmehr and R. Nikandish introduced the **1-absorbing prime ideal**. They defined a non zero proper ideal I of a ring R to be 1-absorbing prime if whenever $a, b, c \in R$ and $abc \in I$, then $ab \in I$ *or* $c \in I$ [6]. The fact that $I \subseteq \sqrt{I}$, for any ideal I implies that the class of 1-absorbing prime ideal is subclass of the class of 1-absorbing primary ideals. Moreover, It is clear from the definition that the 1-absorbing prime also generalizes the 2-absorbing ideals.
   We summarize:

   Prime ideals $\Longrightarrow$ 2-absorbing ideals $\Longrightarrow$ 1-absorbing prime ideals $\Longrightarrow$ 1-absorbing primary ideals

This project **aims** to investigate (some) of the following objectives:

1. Study these generalizations and try to provide new original examples that discriminate between the aforementioned concepts (here one may appeal to ring extensions such as Nagata idealization, amalgamated duplications, …) .
2. Generalize (if possible) the 1-absorbing prime ideals into n- absorbing prime ideals in analogue way to the work done in [1].
3. Try to introduce a weakly versions of the 1-absorbing prime ideals in analogue way to the work done in [3] and [5]
4. Try to lift the 1-absorbing prime concept into other structures such as semi-rings.

References:

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